

6 Draw the following subsets of \mathbb{R}^2 .

6.1 $V = \left\{ \vec{x} \in \mathbb{R}^2 : \vec{x} = \begin{bmatrix} 0 \\ t \end{bmatrix} \text{ for some } t \in \mathbb{R} \right\}$.

6.2 $H = \left\{ \vec{x} \in \mathbb{R}^2 : \vec{x} = \begin{bmatrix} t \\ 0 \end{bmatrix} \text{ for some } t \in \mathbb{R} \right\}$.

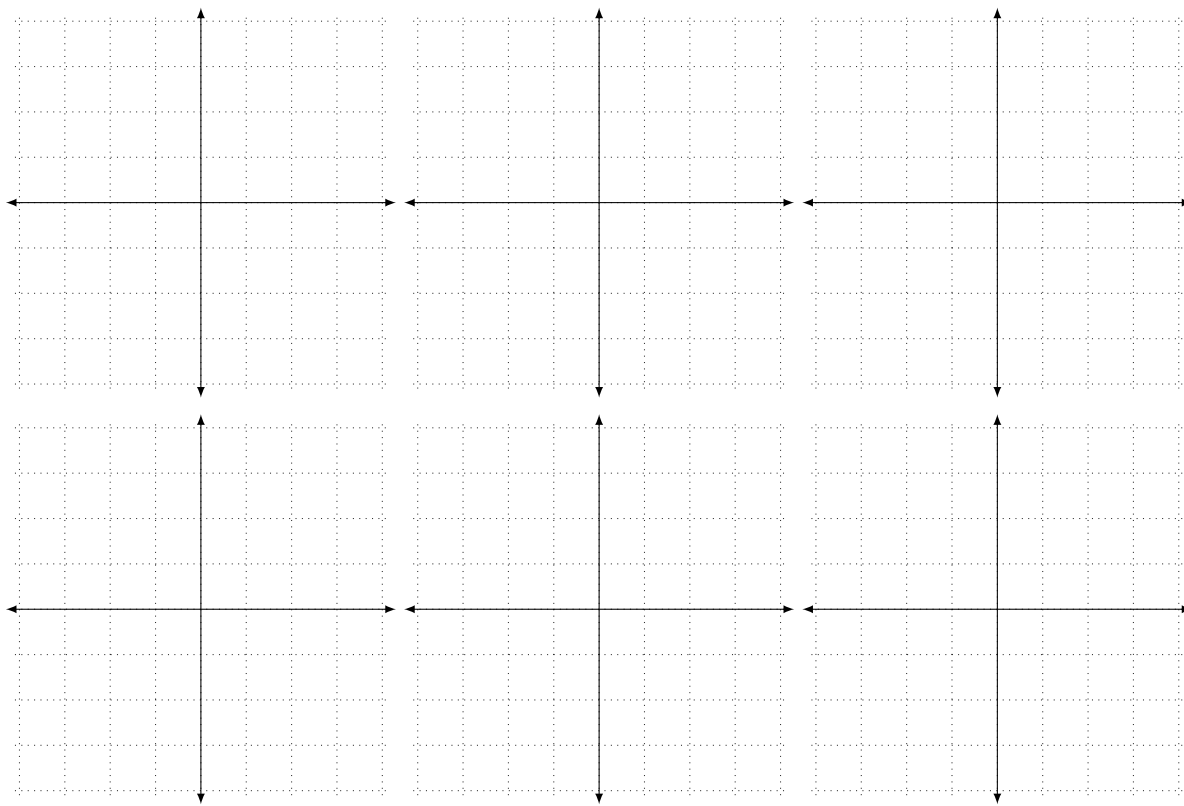
6.3 $D = \left\{ \vec{x} \in \mathbb{R}^2 : \vec{x} = t \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ for some } t \in \mathbb{R} \right\}$.

6.4 $N = \left\{ \vec{x} \in \mathbb{R}^2 : \vec{x} = t \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ for all } t \in \mathbb{R} \right\}$.

6.5 $V \cup H$.

6.6 $V \cap H$.

6.7 Does $V \cup H = \mathbb{R}^2$?



Vector Combinations

Linear Combination

DEFINITION

A **linear combination** of the vectors $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ is a vector

$$\vec{w} = \alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 + \dots + \alpha_n \vec{v}_n.$$

The scalars $\alpha_1, \alpha_2, \dots, \alpha_n$ are called the **coefficients** of the linear combination.

7 Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, and $\vec{w} = 2\vec{v}_1 + \vec{v}_2$.

7.1 Write \vec{w} as a column vector. When \vec{w} is written as a linear combination of \vec{v}_1 and \vec{v}_2 , what are the coefficients of \vec{v}_1 and \vec{v}_2 ?

7.2 Is $\begin{bmatrix} 3 \\ 3 \end{bmatrix}$ a linear combination of \vec{v}_1 and \vec{v}_2 ?

7.3 Is $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ a linear combination of \vec{v}_1 and \vec{v}_2 ?

7.4 Is $\begin{bmatrix} 4 \\ 0 \end{bmatrix}$ a linear combination of \vec{v}_1 and \vec{v}_2 ?

7.5 Can you find a vector in \mathbb{R}^2 that isn't a linear combination of \vec{v}_1 and \vec{v}_2 ?

7.6 Can you find a vector in \mathbb{R}^2 that isn't a linear combination of \vec{v}_1 ?

Recall the *Magic Carpet Ride* task where the hover board could travel in the direction $\vec{h} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and the magic carpet could move in the direction $\vec{m} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

- 8.1 Rephrase the sentence “*Gauss can be reached using just the magic carpet and the hover board*” using formal mathematical language.
- 8.2 Rephrase the sentence “*There is nowhere Gauss can hide where he is inaccessible by magic carpet and hover board*” using formal mathematical language.
- 8.3 Rephrase the sentence “ \mathbb{R}^2 is the set of all linear combinations of \vec{h} and \vec{m} ” using formal mathematical language.